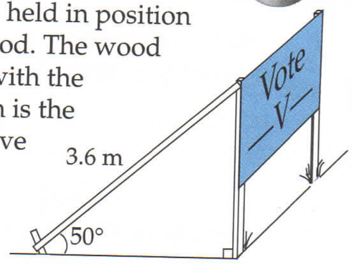
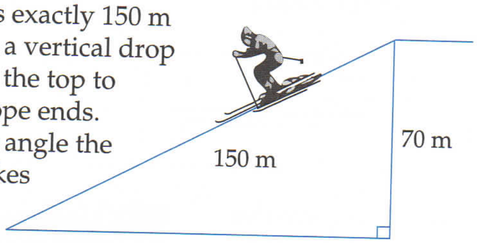
24 Problem-solving using trigonometry and Pythagoras – 2

24.03 Applications of trigonometry – two dimensions

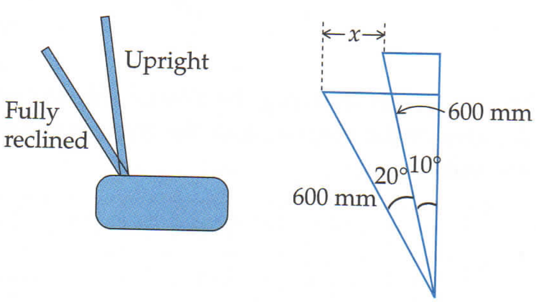
1 An election hoarding is held in position by a 3.6 m length of wood. The wood makes an angle of 50° with the level ground. How high is the top of the hoarding above ground level?



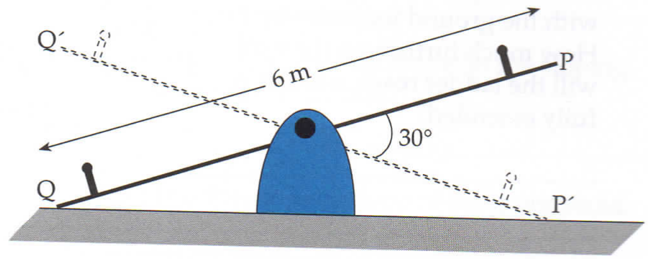
2 A ski-slope is exactly 150 m long and has a vertical drop of 70 m from the top to where the slope ends. Calculate the angle the ski-slope makes with the horizontal.



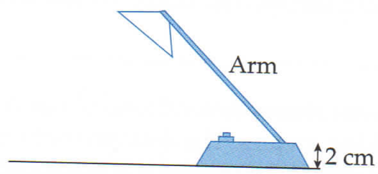
3 The seatback on an aeroplane is 600 mm from the seat to the headrest. In its 'upright' position, the seatback makes an angle of 10° with the vertical. It then reclines through a further angle of 20°. Calculate the length marked x in the diagram (this is how much closer the seatback, when fully reclined, will be to the passenger seated behind).



4 A child's see-saw is exactly 6 m long. It rotates up and down through an angle of 30°. Calculate the height above the ground of one end when the other end is resting on the ground.



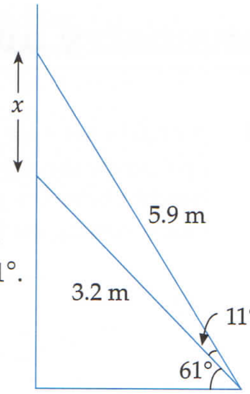
5 The top of a desk-lamp is 46 cm above the desk. The arm makes an angle of 72° with the desk. The base of the lamp is 2 cm high. The length of the arm is x cm.



a Draw a diagram of a right-angled triangle that shows this information.

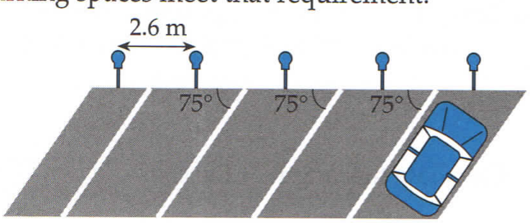
b Calculate the length of the arm.

6 An extension ladder measures 3.2 m at its shortest, and 5.9 m when fully extended. When resting against a wall before being extended, it makes an angle of 61° with the ground. When fully extended, the angle with the ground increases by 11°. How much further up the wall will the ladder reach when it is fully extended?



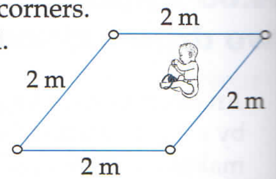
7 The diagram shows several angled car-parks, and their parking meters. The distance between each parking meter is 2.6 m, and the angle between the markings and the footpath is 75°.

The car-parking is supposed to be planned so that a typical large car, with width 195 cm, that is parked in the middle of the space should have a gap of at least 60 cm between it and an adjoining car that is also parked correctly. Decide whether these car-parking spaces meet that requirement.



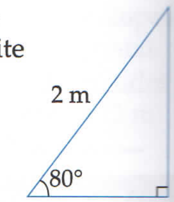
INVESTIGATION
Child's playpen

A child's playpen folds at the corners. It is square when fully opened. The sides measure 2 m.



1 What is the maximum area of the playpen?

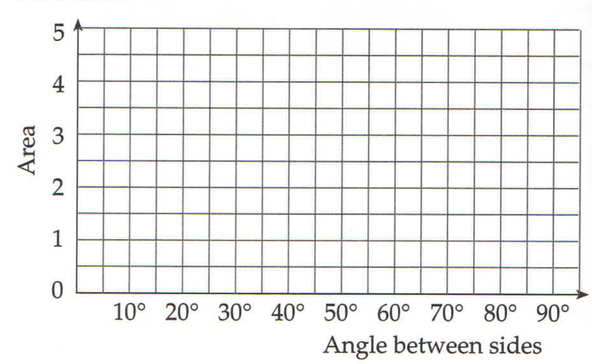
2 When the acute angle between the sides is 80°, the area is 3.94 m². Write some calculations, using trigonometry, to explain why. (Hint: use the triangle shown in the diagram.)



3 Complete this table to show what happens to the area as the playpen is folded. If necessary, use working on another sheet of paper.

Angle between sides	Area (m^2)
90°	
80°	3.94
70°	
60°	
50°	
40°	
30°	
20°	
10°	
0°	

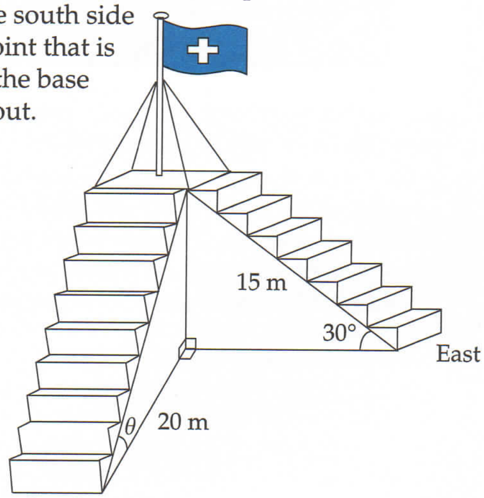
4 Draw a graph showing the relationship between the area of the playpen and the angle between the sides.



24 Problem-solving using trigonometry and Pythagoras – 3

24.04 Applications of trigonometry and Pythagoras – three dimensions

1 People can walk to the top of a lookout by taking steps from the east or south. The steps have vertical sides. On the east side, the steps slope upward at 30° and extend 15 m from the top to the bottom. The steps on the south side start at a point that is 20 m from the base of the lookout.



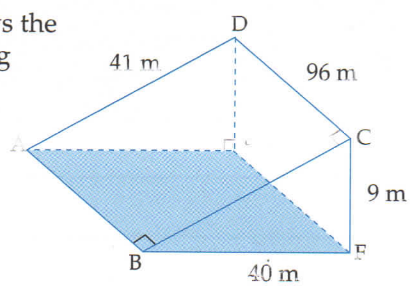
Use trigonometry to:

a calculate the height of the lookout above the ground

b calculate the angle of slope that the steps on the south side make with the ground

c calculate the distance that the steps on the south side extend from the top to the bottom.

2 The diagram shows the model for a sloping grandstand.



a Write some working to show why the length of AF is 104 m.

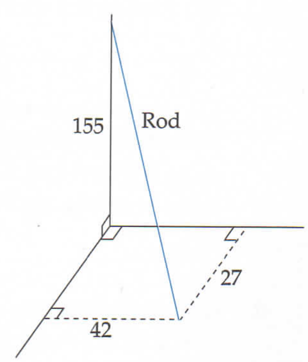
b Name the line of greatest slope on the sloping surface of the grandstand.

c Calculate the angle of slope of the grandstand.

d Add a line to the diagram that would represent the easiest path for wheelchair access from the bottom of the grandstand to the top (this would be the line of least slope).

e Calculate the angle of slope for the path in part **d**.

3 A builder has placed a curtain rod in the corner of a room. The measurements in the diagram are in centimetres.



a Add a line and an angle symbol to show the angle that the rod makes with the floor.

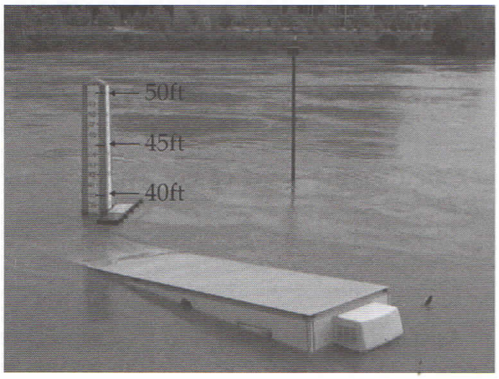
b Calculate the angle in part **a** to the nearest degree.

c Calculate the length of the rod. Round your answer appropriately.

4 Calculate the longest possible length of a billiard cue that can be stored in a cupboard measuring 0.5 m by 0.3 m by 1.8 m.

24.05 Practical measurement applications

1 A flooded river has almost submerged this motorhome. The scale in the background is marked in feet, and shows the height of the water above normal river level. Note: 1 foot is 0.3048 m to 4 significant figures.



a Use the scale in the photo to estimate the height of the water above the normal level. Give your answer to the nearest metre.

b Use a protractor to measure the angle of slope of the roof of the motorhome.

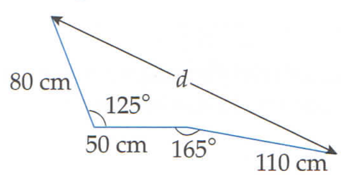
c Use the scale to estimate the vertical distance between the top of the roof of the motorhome at its highest point and the water. Give your answer to the nearest metre.

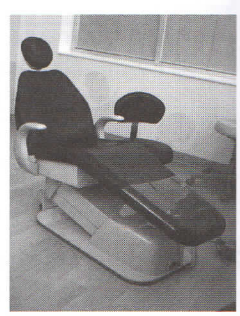
d Use trigonometry and your answers to parts **b** and **c** to estimate the length of the motorhome roof that is above water (this is the distance between the highest edge of the roof and the point where the water is lapping against the roof). Show your working, including a diagram of a right-angled triangle with the measurements placed in the correct positions.

2 The same flood as in question **1** also inundated a car park.

If the distance along the ramp from the top of the ramp to the water is 30 m (to the nearest m), and if the top of the ramp is 4 m above the water level, calculate the angle of slope of the ramp. Show your working, including a diagram of a right-angled triangle with the measurements placed in the correct positions. Give your answer to an appropriate precision.

3 The diagram shows lengths in cm for the three sections of a chair at a dentist's office. The seat is horizontal, and the angles between the seat and seat back and leg rest are shown. Calculate the distance between the top of the seat back and the bottom of the leg rest.





23.03 Angles between lines and planes (page 125)

1 a ∠AEC b 13 m
c 24.8°

2 75.5°

23.04 The angle between two planes (page 126)

1 28.3°

2 a 53.1° b 33.7°

3 a TP b ∠TPM
c 71.6°

23.05 Mixed questions (page 126)

1 a ∠QWU
b ∠SUR
c ∠PVS

2 a $\frac{CF}{CD} = \sin(30°)$
$CF = 12 \times 0.5 = 6$ cm
b First calculate DF:
$\frac{DF}{CD} = \cos(30°)$
$DF = 12 \times 0.8660 = 10.39$ cm
Then $ED^2 = EF^2 + DF^2 = 15^2 + (10.39)^2 = 333$
$ED = \sqrt{333} = 18.25$ cm
c The line of least slope is BD.
$\tan(\angle BDE) = \frac{6}{18.25} = 0.3288$
$\angle BDE = 18.2°$

24 Problem-solving using trigonometry and Pythagoras

24.01 Applications of Pythagoras – two dimensions (page 127)

1 7.36 m

2 1.64 m

3 a 96 cm
b 186 cm

4 Via A, the total distance is $2 \times \sqrt{30^2 + 15^2} + \sqrt{40^2 + 27^2} = 115.34$ m.
Via B, the total distance is $2 \times \sqrt{15^2 + 25^2} + \sqrt{55^2 + 13^2} = 114.83$ m.
B involves less walking – half a metre is saved!

5 244 cm

24.02 Trigonometry – estimation and appropriate precision (page 128)

1 a 170 m
b 168.1 m (or 170 m rounded to nearest 10 m)
c Because the height is given to 2 sf or nearest 10 m only, and the angle measurement is a whole number, the answer should also be given to nearest 10 m only. It is not particularly useful to give this height very accurately.

2 a 79 m (rounded from 79.04)
b An estimate correct to 1 m would be appropriate, and because it is close to a multiple of 10 m, it would be even more appropriate to give the answer as 80 m. The 79 m will be an underestimate of the actual length required, because it has been rounded down from the calculated value but, more importantly, the wire will not be straight – more than even 80 m will be needed because the wire will sag, and some extra length may be needed to secure the wire to both ends.

24.03 Applications of trigonometry – two dimensions (page 129)

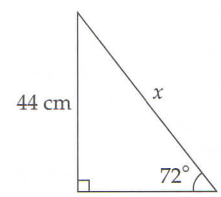
1 2.76 m

2 27.8°

3 196 mm (nearest mm)

4 1.55 m

5 a



b 46.3 cm

6 2.81 m

7 The width of each car-parking space is $2.6 \times \sin(75°)$ = 2.51 m = 251 cm.
The required width is 195 cm + 2 × 30 (30 cm on either side) = 255 cm.
The car-parking spaces are not quite wide enough.

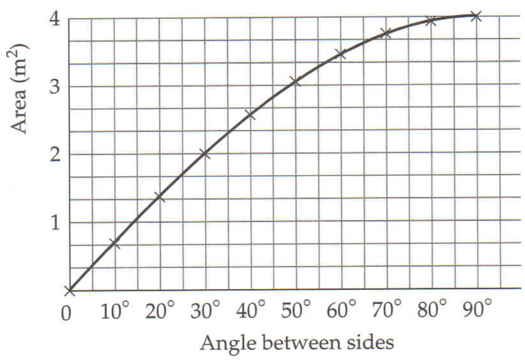
INVESTIGATION Child's playpen (page 130)

1 4 m^2

2 $h = 2 \times \sin 80° = 1.97$ m
Area $= b \times h = 2 \times 1.97 = 3.94$ m^2

3

Angle between sides	Area (m^2)
90°	4
80°	3.94
70°	3.76
60°	3.46
50°	3.06
40°	2.57
30°	2
20°	1.37
10°	0.69
0°	0

4 

24.04 Applications of trigonometry and Pythagoras – three dimensions (page 131)

1 a 7.5 m
b 20.6°
c 21.36 m

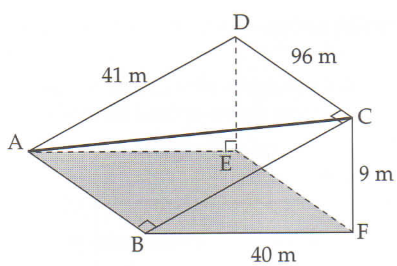
2 a $AF^2 = AB^2 + BF^2$

$= 40^2 + 96^2$

$= 10\,816$

$AF = \sqrt{10\,816} = 104$ (4 sf)

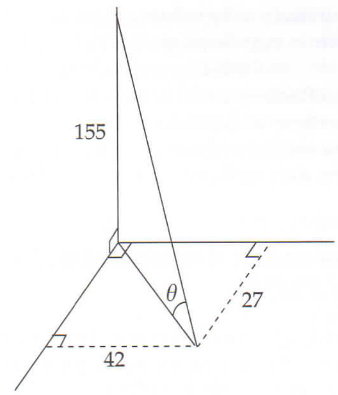
b AD or BC
c 12.7°
d



BD is the other possible answer

e 4.9°

3 a



b 72.1° c 163 cm

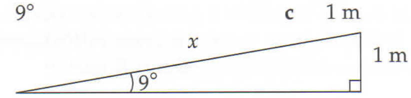
4 1.89 m

24.05 Practical measurement applications (page 132)

1 a Using the given scale the depth is 38 feet. This is 38 × 0.3048 m = 11.58 m or 12 m to the nearest m.

b 9° c 1 m

d



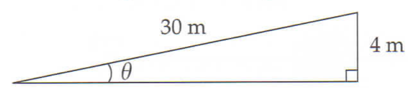
$\frac{1}{x} = \sin(9°)$

$x = \frac{1}{\sin(9°)}$

$= 6.39$

An appropriate estimate would be 6 m, given the inaccuracy in using the scale and protractor.

2



$\sin(\theta) = \frac{4}{30} = 0.1333$

$\theta = 7.6°$

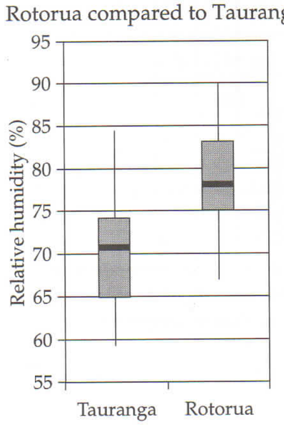
An appropriate estimate would be 8°.

3 222.9 cm (or 223 cm to nearest cm)

25 Multivariate datasets and the statistical enquiry cycle

25.01 Analysing multivariate data – 2 (page 135)

1 The graph shows comparative boxplots. Clearly, the two boxes do not overlap, and therefore we can make the claim that the samples show there tends to be significantly higher relative humidity in Rotorua compared to Tauranga.



We can also calculate the quartiles and medians for both sets of sample data.

	Tauranga	Rotorua
Minimum	59.3	67
LQ	64.6	75
Median	70.7	78
UQ	73.95	83
Maximum	84.4	90

Difference between the medians (DBM) 7.3
Overall visible spread (OVS) 18.4
DBM as a percentage of OVS 40%

The DBM is $\frac{2}{5}$ of the OVS (considerably more than $\frac{1}{3}$) so, again, we can make the claim that the samples show there tends to be significantly higher relative humidity in Rotorua compared to Tauranga.

11 Algebraic fractions – 1

NCEA Mathematics and Statistics AS 1.2

Apply algebraic methods

Achievement	Achievement with *Merit*	Achievement with *Excellence*
Apply algebraic methods	Apply algebraic methods involving relational thinking	Apply algebraic methods involving extended abstract thinking

Level 6

- **A01 Generalise the properties of operations with rational numbers, including the properties of exponents**

11.01 Simplification

Algebraic fractions are **simplified** by looking for a common factor in both the numerator and denominator and 'cancelling' it.

Examples

$\frac{12ab}{18ac} = \frac{2b}{3c}$ (the common factors were 6 and a)

$\frac{3x^3y}{6xy^2} = \frac{x^2}{2y}$ (the common factors were 3, x and y)

1–5 Simplify these algebraic fractions.

1 $\frac{4x}{12y}$ ______

2 $\frac{6ab}{3c}$ ______

3 $\frac{a}{2a}$ ______

4 $\frac{12p}{9}$ ______

5 $\frac{x^4}{x}$ ______

6–12 Simplify these algebraic fractions.

6 $\frac{5x^2}{x^3}$ ______

7 $\frac{6x}{12x^2}$ ______

8 $\frac{4xy^2}{3x^2z}$ ______

9 $\frac{30x^2y}{24xy}$ ______

10 $\frac{12x^2yz^4}{16xy^2z^3}$ ______

11 $\frac{(2x^2)^2}{4x}$ ______

12 $\frac{(8x^3)^2}{12x^4}$ ______

11.02 & 11.03 Multiplication and division of algebraic fractions

Algebraic fractions follow the same rules as ordinary fractions when multiplying and dividing.

- To **multiply**, multiply the numerators together to get the new numerator. Do the same for the denominator. Then simplify if possible.
- To **divide**, multiply the first fraction by the reciprocal of the second fraction.

Examples

Multiplying: $\frac{4x}{5y} \times \frac{15x^2y}{2} = \frac{60x^3y}{10y}$

$= 6x^3$

Dividing: $\frac{4x}{3} \div \frac{6}{x^2} = \frac{4x}{3} \times \frac{x^2}{6}$

$= \frac{4x^3}{18}$

$= \frac{2x^3}{9}$

1–5 Multiply these pairs of fractions. Write your answer in its simplest possible form.

1 $\frac{5x}{y} \times \frac{2x}{y}$ ______

2 $\frac{3x}{2y} \times \frac{4x}{5y}$ ______

3 $\frac{4x^2}{3} \times \frac{2}{x}$ ______

4 $\frac{6}{x} \times \frac{2}{3x^2}$ ______

5 $\frac{2x^2}{3y} \times \frac{6y^2}{x}$ ______

6–10 Divide these pairs of fractions. Write your answer in its simplest possible form.

6 $\frac{4x}{y} \div \frac{6x}{y}$ ______

7 $\frac{2x}{3} \div \frac{4x}{9}$ ______

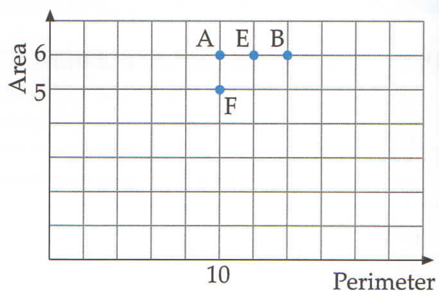
8 $\frac{6x^2}{5} \div \frac{15}{x}$ ______

9 $\frac{x^4}{6} \div \frac{x}{2}$ ______

10 $\frac{12x^2y^3}{xy} \div \frac{2y^2}{3x}$ ______

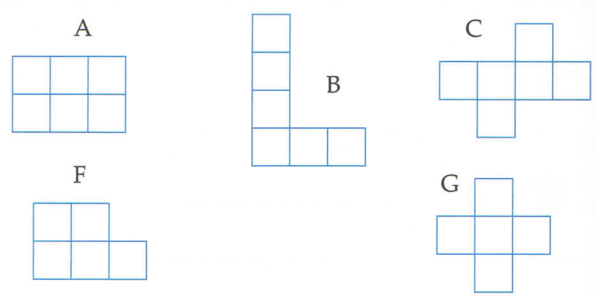
INVESTIGATION The relationship between area and perimeter for polyominoes

Polyominoes are shapes made up of the same size squares joined side to side. This graph shows the relationship between area and perimeter for some pentominoes and hexominoes. Each shape is made up of 1 cm by 1 cm squares.



1 Add a label on this graph to represent C.

2 Draw a possible hexomino that could be represented by point E.



3 Add a point and label to the graph to represent G.

4 Add two points (D and T) to the graph to represent a domino (two squares) and a tromino (three squares).

5 What does the graph show about the general relationship between the perimeter and area of polyominoes?

______

______

______

______

11 Algebraic fractions – 2

11.04 & 11.05 Common factors

Some fractions have common factors in both the numerator and denominator. These fractions can be simplified by 'cancelling' the common factors.

Example

$$\frac{2x(x-2)}{8(x-2)} = \frac{2x}{8} = \frac{x}{4}$$

Sometimes you need to factorise the numerator and/or denominator first.

Examples

$$\frac{8x+4}{6x+3} = \frac{4(2x+1)}{3(2x+1)} = \frac{4}{3}$$

$$\frac{x^2-4}{x^2+x-2} = \frac{(x+2)(x-2)}{(x+2)(x-1)} = \frac{x-2}{x-1}$$

(No further simplifying is possible.)

Simplify these algebraic fractions. You may need to factorise one or both of the numerator and denominator first.

1 $\frac{6(x-y)}{16}$

2 $\frac{5(x-1)}{7(x-1)}$

3 $\frac{4x(6x+1)}{2}$

4 $\frac{5x-25}{3x-15}$

5 $\frac{x(x+2)}{(x-1)(x+2)}$

6 $\frac{2x+4}{2}$

7 $\frac{2x-4}{x^2-4}$

8 $\frac{x^3}{x^2-x}$

9 $\frac{x^2+5x+6}{x^2+6x+8}$

10 $\frac{x-1}{x^2-1}$

11 $\frac{4x^2+12xy}{8x^3}$

11.06 Addition – same denominator

Fractions can be added directly if they have the same denominator. The numerators are added (or subtracted) and the result placed over the denominator. Then simplify this fraction if possible.

Example

$$\frac{11x}{12} - \frac{7x}{12} = \frac{11x-7x}{12} = \frac{4x}{12} = \frac{x}{3}$$

Add or subtract these fractions. Write your answer as simply as possible.

1 $\frac{2x}{9} + \frac{5x}{9}$

2 $\frac{9x}{10} - \frac{3x}{10}$

3 $\frac{5x}{6} + \frac{x}{6}$

4 $\frac{x}{5} + \frac{x}{5}$

5 $\frac{6}{x} + \frac{8}{x}$

6 $\frac{11y}{12} - \frac{5y}{12}$

7 $\frac{3x}{y} + \frac{2x}{y}$

8 $\frac{2x^2}{3} - \frac{x^2}{3}$

11.07 Addition – different denominators

Fractions can only be added/subtracted when they are written with the same denominator. This **common denominator** can be the lowest common multiple of the two denominators.

Example

Add $\frac{x}{8} + \frac{5x}{12}$.

Answer

A suitable common denominator is 24, so we rewrite both fractions with a denominator of 24.

$$\frac{x}{8} + \frac{5x}{12} = \frac{3x}{24} + \frac{10x}{24} = \frac{13x}{24}$$

We can also use 'cross-multiplying'. Here the common denominator is worked out by multiplying the two denominators, and the new numerators are the result of multiplying by the denominator diagonally opposite.

$$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$$

$a \quad c$
$b \quad d$

Example

$$\frac{2x}{3} - \frac{x}{5} = \frac{10x - 3x}{15} = \frac{7x}{15}$$

Work out the answers to these addition and subtraction questions. Write your answers as simply as possible.

1 $\frac{x}{4} + \frac{x}{5}$

2 $\frac{x}{2} - \frac{x}{8}$

3 $\frac{3x}{5} + \frac{2x}{3}$

4 $\frac{5x}{6} - \frac{3x}{4}$

5 $\frac{7x}{12} + \frac{7x}{24}$

6 $\frac{3x}{2} - \frac{x}{4}$

7 $\frac{2x^2}{3} + \frac{x^2}{6}$

8 $\frac{x+1}{2} + \frac{x-4}{3}$

9.06 Simple exponential equations – unknown in the power (page 46)

1 a $x=7$ b $x=6$
c $x=4$ d $x=2$
e $x=5$ f $x=4$

2 a $x=3$ b $x=8$
c $x=6$ d $x=5$
e $x=5$ f $x=6$

3 a $x=6$ b $x=2$
c $x=8$ d $x=14$

PUZZLE **Who goes to Mount Maunganui? (page 46)**

The lawyer

10 Simultaneous equations

10.01 Elimination method (page 47)

1 $x=4$ and $y=6$

2 $x=16$ and $y=5$

3 $x={}^{-}1$ and $y=3$

4 $x=\frac{1}{2}$ and $y=8$

INVESTIGATION **Treasures Baby Club points (page 47)**

1 Two 5-point tokens and one 6-point token

2 No, no

3 19 points

10.02 Subtracting equations to eliminate (page 48)

1 $x=4$ and $y=3$

2 $x=5$ and $y=6$

3 $x=7$ and $y={}^{-}1$

4 $x=6$ and $y=\frac{{}^{-}1}{2}$

10.03 Adjusting coefficients – one equation (page 48)

1 $x=2$ and $y=6$

2 $x=8$ and $y={}^{-}1$

3 $x=10$ and $y={}^{-}2$

4 $x=\frac{1}{2}$ and $y={}^{-}3$

10.03 Adjusting coefficients – two equations (page 49)

1 $x=0$ and $y=5$

2 $x=9$ and $y={}^{-}2$

10.04–10.06 Substitution method (page 49)

1 $x=2$ and $y=9$

2 $x=10$ and $y=2$

3 $x=0$ and $y={}^{-}1$

4 $x={}^{-}3$ and $y=10$

5 a $x+30+11y=180$
$2x+y+90=180$
These simplify:
$x+11y=150$
$2x+y=90$

b $x=40$ and $y=10$

c 70°, 80°, 110°, 100°

6 a $2x+11=5y$
$2x=4y-2$

b $x=17$ and $y=9$

c 45°, 34°, 101°, 45°, 34°, 101°

11 Algebraic fractions

11.01 Simplification (page 51)

1 $\frac{x}{3y}$

2 $\frac{2ab}{c}$

3 $\frac{1}{2}$

4 $\frac{4p}{3}$

5 x^3

6 $\frac{5}{x}$

7 $\frac{1}{2x}$

8 $\frac{4y^2}{3xz}$

9 $\frac{5x}{4}$

10 $\frac{3xz}{4y}$

11 x^3

12 $\frac{16x^2}{3}$

11.02 & 11.03 Multiplication and division of algebraic fractions (page 51)

1 $\frac{10x^2}{y^2}$

2 $\frac{6x^2}{5y^2}$

3 $\frac{8x}{3}$

4 $\frac{4}{x^3}$

5 $4xy$

6 $\frac{2}{3}$

7 $\frac{3}{2}$

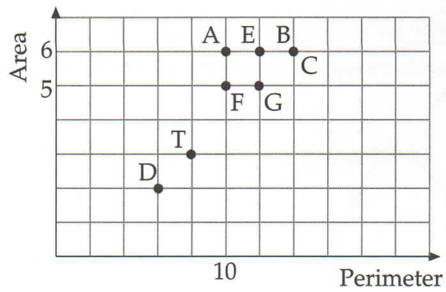
8 $\frac{2x^3}{25}$

9 $\frac{x^3}{3}$

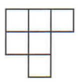
10 $18x^2$

INVESTIGATION The relationship between area and perimeter for polyominoes (page 52)

1, 3, 4



2



5 In general, as the area increases the perimeter increases.

11.04 & 11.05 Common factors (page 53)

1 $\frac{3(x-y)}{8}$

2 $\frac{5}{7}$

3 $2x(6x+1)$

4 $\frac{5}{3}$

5 $\frac{x}{x-1}$

6 $x+2$

7 $\frac{2}{x+2}$

8 $\frac{x^2}{x-1}$

9 $\frac{x+3}{x+4}$

10 $\frac{1}{x+1}$

11 $\frac{x+3y}{2x^2}$

11.06 Addition – same denominator (page 53)

1 $\frac{7x}{9}$

2 $\frac{3x}{5}$

3 x

4 $\frac{2x}{5}$

5 $\frac{14}{x}$

6 $\frac{y}{2}$

7 $\frac{5x}{y}$

8 $\frac{x^2}{3}$

11.07 Addition – different denominators (page 54)

1 $\frac{9x}{20}$

2 $\frac{3x}{8}$

3 $\frac{19x}{15}$

4 $\frac{x}{12}$

5 $\frac{7x}{8}$

6 $\frac{5x}{4}$

7 $\frac{5x^2}{6}$

8 $\frac{5(x-1)}{6}$

12 Number and spatial patterns

12.01 Numerical substitution (page 55)

1 a 17 b −7
c 32 d 6
e 27 f 3
g Undefined h 65

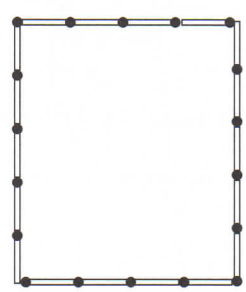
2 a −20 b −3
c 1 d 16
e −25 f 3

3 a i −200
ii 200
iii Undefined
iv 280
v 580
b Counting numbers – that is, 1, 2, 3, 4, …
c $600

4 a Vinh: 10.5 mL
b George: 55 mL

12.02 Linear patterns (page 56)

1 a



b

Length of base, x	Total number of matchsticks, n
1	6
2	10
3	14
4	18
…	…
15	62
25	102

c $n = 4x + 2$